Sectoral Heterogeneity, the Skill Premium and Productivity Dynamics

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Abstract

We present a general equilibrium dynamic model, which focuses on how the heterogeneity in sector productivity affects the macroeconomic equilibrium. In the model, individuals endogenously choose whether to invest in their education and work in the advanced sector or not, and firms endogenously choose in which sector to operate and to invest their capital. The resulting macroeconomic equilibrium exhibits a feedback between investment in education and investment of physical capital in the advanced sector, as one promotes the marginal productivity of the other. This feedback yields several results: (i) the skill premium rises over time; (ii) along the transitional dynamics both TFP and labor productivity increase; (iii) income inequality may increases over time, but may also display a Kuznets curve pattern.

Keywords: Income Distribution, Economic Growth, Factor Productivity

1. Introduction

We present a general equilibrium dynamic model with two sectors – one more productive than the other. We use this model to analyze how the heterogeneity in sector productivity affects the dynamics of labor productivity, total factor productivity, skill premium, income inequality and physical and human capital accumulation.

In the model, in order to work in the more productive sector, an individual has to acquire education. Acquiring education is an individual choice based on expected future skill premium and on the cost of education. Firms choose their technology endogenously, as they have to choose between operating in

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the more advanced sector or in the less advanced one. The analysis of the resulting macroeconomic equilibrium highlights a feedback relationship between investment in education and investment of physical capital in the advanced sector, as one promotes the marginal productivity of the other. This feedback mechanism leads to one of our main results, namely that the skill premium rises over time.

We also find that the rising skill premium may lead to a dynamic pattern of a rising income inequality. However, it is also possible that income inequality shall not be monotonically rising, but instead, shall exhibit Kuznets curve dynamics, in which it initially rises and from a certain point in time begins to decline, even though the skill premium continues to rise. The decline in inequality occurs when the number of high-skilled workers is sufficiently large to make the relative equality within the high-skilled workers dominate the rising inequality between the two groups of workers.

The increasing share of physical capital allocated to the advanced sector, and the rise in the share of the population that chooses to become high-skilled, make both labor productivity and TFP rise over time. Hence, our paper highlights an indirect channel through which investment in human capital promotes economic growth. The direct channel is based on the result that having more human capital leads to larger production even if the allocation of physical capital remains the same. The indirect channel is based on the property that having more human capital also attracts a larger share of the economy's physical capital to the more advanced sector.[2] These results about the rise in the skill premium, income inequality and productivity fit the empirical findings presented by a massive body of literature.

Our paper is related to several strands of the literature. First, in the past few decades, many studies have argued that differences in output per capita between countries stem from differences in productivities. Productivity differences between countries were explained by either technological differences (e.g. Romer (1993)), or other, non-technological, differences such as capital barriers (e.g. Restuccia (2004) and Parente et al. (2000)), or different institutional and governmental infrastructure (e.g. Hall and Jones (1999)).[3] Our paper adds another explanation to this branch of the literature –an explanation that is based on the endogenous choices of firms and individuals with regard to technology and factor allocation between the less advanced and more advanced sectors.

In dealing with productivity dynamics, with an emphasis of the role of sectoral heterogeneity, our paper is particularly close in its nature to Acemoglu and Zilibotti (2001) and Caselli and Coleman (2006). Acemoglu and Zilibotti (2001) argue that the mismatch between technologies and human capital en-

[2] This result is close in its nature to the result in Zeira (2009). In his model, an increase of the stock of educated workers increases the profitability of adopting a new type of machines, and thus promotes economic growth indirectly.

[3] Another explanation for TFP differences relies on misallocation of factors of production between heterogeneous firms. For a survey of this literature see Restuccia and Rogerson (2013).

dowments yields productivity differences between countries. As in Acemoglu and Zilibotti (2001), our results spring from a mechanism in which with more high-skilled labor, the advanced sector attracts a larger magnitude of investment, either in physical capital (as in our model) or in R&D (as in Acemoglu and Zilibotti (2001)), which augments the productivity of this sector. One of the main differences between our study and theirs is that in our model human capital is endogenous. Another difference is that we analyze the transition towards the steady state, rather than focus on the steady state of a balanced growth path. Caselli and Coleman (2006) find empirically that countries with higher human capital endowment choose more skill intensive technologies than countries where human capital is scarce do. Unlike our paper, Caselli and Coleman (2006) do not focus on individuals choices and also do not focus on the dynamics of productivity and inequality.

Our study also relates to the vast literature about the interrelation between human capital acquisition, the dynamics of the skill premium and income inequality trends. It is a well documented fact that the skill premium has risen in the past decades despite the large increase in the stock of educated workers.[4] It is also known that inequality has risen during these decades (See, for example, Autor et al. (2008) for evidence of the rising income inequality in the United States since 1980). These two phenomena took place while the supply of skills rose as well. [5] In contrast to our explanation for these facts, most articles in this literature provide explanations based on either skill biased technical change (e.g. Galor and Moav (2000)), or capital-skill complementarity (e.g. Krusell et al. (2000)).

2. The Model

Consider a closed OLG economy with a constant population along time. Each generation lives three periods. In the first period all individuals are learners, as they may acquire higher education; In the second period, all individuals work according to their educational level, consume, save and give birth to one offspring; In the third period of life all individuals are retirees, and consume all their savings. Production takes place according to two production processes: less advanced and more advanced. In order to work in the advanced sector, individuals have to acquire education, which is costly; Firms, too, have to decide in which sector to invest. All markets are fully competitive, and therefore factor prices equal their marginal product.

2.1. Production and Factor Prices

Production takes place in a fully competitive environment. Aggregate output at period t, Y_t, is produced by two technologies, low-skilled intensive and high-

[4]Krusell et al. (2000) provide compelling evidence that the skill premium has risen dramatically from 1980.

[5]The rise in the supply of skills is well documented as well. See, for example, Goldin & Katz (2007).

skilled intensive, L and H, respectively:

$$Y_t = A_H \left(K_t^H\right)^{\alpha} H_t^{1-\alpha} + A_L \left(K_t^L\right)^{\alpha} L_t^{1-\alpha} = A_H H_t \left(k_t^H\right)^{\alpha} + A_H L_t \left(k_t^L\right)^{\alpha}, \quad (1)$$

where $A_H > A_L$ are sector specific technology parameters, K_t^O is the capital employed in sector $O \in \{H, L\}$ at period t; L_t and H_t are the stocks of high-skilled- and low-skilled- labor that are employed in production respectively; and $k_t^O \equiv \frac{K_t^O}{O_t}$.

2.1.1. Factor Prices

Factor markets are competitive, and therefore factor prices equal their marginal product:

$$R_t = \alpha A_H \left(k_t^H\right)^{\alpha-1} = \alpha A_L \left(k_t^L\right)^{\alpha-1}, \quad (2)$$

and the inverse demand for each type of workers is given by:

$$w_t^H = (1-\alpha) A_H \left(k_t^H\right)^{\alpha}, \quad (3)$$

and

$$w_t^L = (1-\alpha) A_L \left(k_t^L\right)^{\alpha}, \quad (4)$$

where R_t is the rental rate of physical capital and w_t^O is the wage paid at period t for worker of type O.

2.2. Individuals

Individuals derive utility from consumption in their second and third periods of life. For simplicity, we assume that the utility function takes the following form:

$$u^i(c_t^i, c_{t+1}^i) = (1-\beta)\ln(c_t^i) + \beta \ln(c_{t+1}^i), \quad (5)$$

where $\beta \in (0, 1)$ and c_t^i is the consumption of individual i at period t. Each individual faces a budget constraint:

$$c_t^i + \frac{c_{t+1}^i}{R_{t+1}} \leq W_t^i, \quad (6)$$

where W_t^i is the wealth of individual i at period t. The wealth of individual i depends on his educational level, and on his individual educational cost (if he decided to acquire higher education). We assume that individuals in each generation are heterogenous in ability. The heterogeneity is materialized in the cost of acquiring higher education, h_t^i –the higher the ability the lower the cost. We assume that this cost is uniformly distributed in the range $(0, 1)$ and i.i.d. across generations. Hence, the wealth of individual i is given by:

$$W_t^i = \begin{cases} w_t^L & \text{if } i \text{ is an low-skilled worker} \\ w_t^H - h_{t-1}^i \cdot R_t & \text{otherwise.} \end{cases}$$

3. Savings and Capital Investment

3.1. Individual's Optimization Solution

Each individual lives three periods. In the first period each individual decides whether to acquire higher education and become a high-skilled worker in the second period of life, or give up education and become a low-skilled worker. In the second period each individual supplies inelastically his unique unit of time to the labor market, according to his educational level; he consumes, gives birth to one offspring and saves for his consumption in his retirement period. Hence, in the second period of life each individual has to divide his wealth between consumption in the second and third periods of life. Since the utility function is separable, we can first analyze this last decision, and then, based on this decision we analyze backwards the educational decision.

3.1.1. Consumption -Savings Decision

Given his educational level, in his second period of life, individual i chooses c_t^i and s_t^i so as to maximize his utility as given by (5), under the budget constraint as given by (6) and $s_t^i = \frac{c_{t+1}^i}{R_{t+1}}$. It is straightforward that $c_t^{i*} = (1-\beta)W_t^i$, and $s_t^{i*} = \beta W_t^i$. Note that given the educational level, the wealth of individual i is uniquely determined, since it is the sum of the income he receives as a worker, net of his educational expenses, if he decided to acquire higher education in his first period of life. Hence, the consumption level in both second and third periods of life are uniquely determined.

3.1.2. Educational Decision

At the first period of life, individual i decides whether to acquire education. Clearly, individual i acquires education if his utility is higher as a skilled worker. Acquiring education entails a cost, h_t^i. Since $c_{t+1}^i = (1-\beta)W_{t+1}^i$ and $c_{t+2}^i = \beta W_{t+1}^i R_{t+2}$, the indirect utility of individual i from his wealth is given by:

$$V^i(W_{t+1}) = \ln[(1-\beta)W_{t+1}^i] + \ln(\beta W_{t+1}^i R_{t+2}). \tag{7}$$

It is straightforward that the higher the wealth the higher the (indirect) utility. As a result, individual i acquires education if (and only if):

$$w_{t+1}^H - R_{t+1}h_t^i \geq w_{t+1}^L$$

This in turn yields a cost threshold, $\overline{h}_t$:

$$\overline{h}_t = \frac{w_{t+1}^H - w_{t+1}^L}{R_{t+1}}, \tag{8}$$

below which all individuals acquire education, and above which individuals do not acquire education.

3.2. Physical Capital Allocation Decision

It is straightforward from (2) that

$$A_H \left(k_t^H\right)^{\alpha-1} = A_L \left(k_t^L\right)^{\alpha-1},$$

which means that in equilibrium, the marginal product of capital is equalized in both sectors. This last equation implies:

$$k_t^H = \gamma k_t^L, \tag{9}$$

where $\gamma \equiv (A_H/A_L)^{\frac{1}{1-\alpha}}$. This last equation implies that in equilibrium, the higher the ratio of productivities in the two sectors, the higher the ratio of capital per worker in the two sectors. This equation also implies that the higher the ratio of high-skilled- to low-skilled-labor, the higher the ratio of physical capital allocated in the skilled sector to the unskilled sector (K_t^H/K_t^L). In order to see this, let us substitute the definitions of k_t^O into the last equation. Clearly, the higher H_t (and thus the lower L_t), the higher the ratio K_t^H/K_t^L.

4. The Dynamical System

4.1. The Dynamics of the Labor Force

Recall that ability is uniformly distributed in the range $(0,1)$. Recall also that at each period t all individuals with ability lower than $\overline{h}_t$ acquire higher education, whereas the rest of the generation forms the low-skilled labor force. This in turn implies that the supply of skilled labor at period $t+1$ is given by:

$$H_{t+1} = \overline{h}_t, \tag{10}$$

and the supply of unskilled labor is given by:

$$L_{t+1} = 1 - \overline{h}_t. \tag{11}$$

4.2. Physical Capital Formation

Physical capital for period $t+1$ is formed during period t, and satisfies (9). Note that the funds for financing the formation of physical capital stem from the aggregate savings in the economy. The aggregate savings, $S_t = \sum_i s_t^i = \beta(1-\alpha)Y_t - R_t \int_0^{\overline{h}_{t-1}} h_{t-1}^i \cdot f(h_{t-1}^i)di = \beta(1-\alpha)Y_t - 1/2 R_t \overline{h}_{t-1}^2$. This implies that the aggregate amount of capital at period $t+1$ which is allocated in the two sectors accompanied by the investment in human capital, which equals $\int_0^{\overline{h}_t} h_t^i f(h_t^i)di = \frac{1}{2}\overline{h}_t^2$, must equal the savings of period t:

$$K_{t+1}^H + K_{t+1}^L + \frac{1}{2}\overline{h}_t^2 = \beta\left[(1-\alpha)Y_t - \frac{1}{2}R_t\overline{h}_{t-1}^2\right] \tag{12}$$

Using (10) and the definitions of k_t^H and k_t^L, this last equation becomes:

$$\frac{1}{2}(H_{t+1})^2 + H_{t+1}\left[k_{t+1}^L(\gamma-1)\right] + k_{t+1}^L - \beta\left[(1-\alpha)Y_t - \frac{1}{2}R_t\overline{h}_{t-1}^2\right] = 0. \tag{13}$$

We will show below that both Y_t and k_t^L can be expressed as a function of H_t solely, and therefore this last equation becomes an autonomous first order dynamic equation. Next, using (8) and (10) we receive:

$$H_{t+1} = \frac{w_{t+1}^H - w_{t+1}^L}{R_{t+1}}. \tag{14}$$

Substituting into it the wages yields:

$$H_{t+1} = \frac{1-\alpha}{R_{t+1}} \cdot \left[A_H \left(k_{t+1}^H\right)^\alpha - A_L \left(k_{t+1}^L\right)^\alpha \right].$$

Using (2) and (9), it is straightforward that the last expression becomes:

$$H_{t+1} = \frac{1-\alpha}{\alpha} \cdot (\gamma - 1)\, k_{t+1}^L,$$

which in turn yields:

$$k_{t+1}^L = \frac{\alpha}{(1-\alpha)(\gamma-1)} H_{t+1}. \tag{15}$$

This last equation implies that output can be represented as a function of H_t alone, using (10), (11), (15) and (9):

$$Y_t = \Gamma H_t^\alpha \left[H_t(\gamma - 1) + 1\right], \tag{16}$$

where $\Gamma \equiv A_L \left[\frac{\alpha}{(1-\alpha)(\gamma-1)}\right]^\alpha$. Next, substituting k_{t+1}^L into (13) yields the following autonomous first-order dynamic equation:

$$H_{t+1} = \frac{-\frac{\alpha}{\gamma-1} + \sqrt{\left(\frac{\alpha}{\gamma-1}\right)^2 + 2\beta\Gamma(1+\alpha)(1-\alpha)^2 H_t^\alpha \left[\frac{1}{2}H_t(\gamma-1)+1\right]}}{1+\alpha}. \tag{17}$$

Definition 1. *A Dynamic Equilibrium in this economy is s sequence of factor prices, factor stocks, ability threshold, output, consumption and savings,* $\{w_t^H, w_t^L, R_t, H_t, L_t, K_t^H, K_t^L, \overline{h}_t, Y_t, C_t, S_t\}_0^\infty$*, such that:*
(a) All individuals choose their educational level, consumption and savings so as to maximize their utility as given by (5), subject to the budget constraint as given by (6);
(b) Factor prices are set as to clear the factor markets, according to (3), (4) and (2); and
(c) Output is determined according to (1);

Lemma 1. *There is a unique dynamic equilibrium in this economy.*

Proof:. Given next period's factor prices, $\overline{h}_t$, c^i_{t+1} and s^i_{t+1} are uniquely determined, as described in (8). Hence, condition (a) in Definition 1 is satisfied. Furthermore, this decision uniquely determines the human capital stocks, as described in (10) and (11). Next, k^L_{t+1} and k^H_{t+1} are uniquely determined, given H_{t+1}, and hence condition (b) is satisfied. Finally, factor prices are uniquely determined by (2), (3) and (4), so factor markets clear. Since all factor stocks are uniquely determined, so is output, as given by (1). □

4.3. The Evolution of Human Capital

Denote $\Delta(H_t) \equiv 2\beta\Gamma(1+\alpha)(1-\alpha)^2 H_t^{\alpha}\left[\frac{1}{2}H_t(\gamma-1)+1\right]$. In order to analyze the dynamics of the economy, we first establish a lemma that shows that $\Delta''(H_t) < 0$.

Lemma 2. $\forall H_t > 0, \Delta''(H_t) < 0$.

Proof. In Appendix A we show that any function of the type $Y = A \cdot X^{1+\alpha} + B \cdot X^{\alpha}$ has a negative second derivative, $\forall X > 0$, and therefore so does the function $\Delta(H_t)$. □

Next, we show in the next lemma that the corresponding equilibrium converges into a unique steady state.

Lemma 3. *The corresponding equilibrium converges into a unique steady state.*

Proof. Differentiating (17) w.r.t. H_t yields:

$$\frac{\partial H_{t+1}}{\partial H_t} = \frac{\Delta'(H_t)}{2(1+\alpha)\sqrt{\frac{\alpha^2}{(\gamma-1)^2} + \Delta(H_t)}} > 0.$$

This expression is positive, because both nominator and denominator are positive (it is straightforward that $\Delta' > 0$). According to Lemma 2, $\Delta'' < 0$. Consequently, the second derivative of the dynamic equation as given below is negative:

$$\frac{\partial^2 H_{t+1}}{\partial (H_t)^2} = \frac{\Delta''(H_t) \cdot 2(1+\alpha)\sqrt{\cdot} - \Delta'(H_t)\frac{\partial\sqrt{\cdot}}{\partial H_t}}{\sqrt{\cdot}^2}.$$

Due to Lemma 2, the nominator is negative. Since the denominator is positive, the second derivative is negative. Hence, the dynamic equation converges to a unique steady state. □

4.4. Steady State analysis

In the steady state, $H_{t+1} = H_t$, which implies that all other variables are constant as well. Using (17) and the property that $H_{t+1} = H_t = H^*$, we receive:

$$H^* = \frac{-\frac{\alpha}{\gamma-1} + \sqrt{\left(\frac{\alpha}{\gamma-1}\right)^2 + 2(1+\alpha)(1-\alpha)\beta\Gamma H^{*\alpha}\left[H^*(\gamma-1)+1\right]}}{1+\alpha},$$

when substituting output into it. Rearranging this last equation yields:

$$(1+\alpha)(H^*)^{2-\alpha} + \frac{2\alpha}{\gamma-1}(H^*)^{1-\alpha} - \beta\Gamma(1-\alpha)^2[(\gamma-1)H^*+2] = 0. \qquad (18)$$

This equation constructs an implicit function of H^* as a function of all parameters. Our numerical analysis shows that $\frac{\partial H^*}{\partial \beta} > 0$, $\frac{\partial H^*}{\partial A_H} > 0$ and $\frac{\partial H^*}{\partial A_L} > 0$. The intuition for these results is straightforward: A higher value of β implies that in each period the savings are higher and therefore so is investment in both physical and human capital. The higher growth rate of skilled labor accelerates also the allocation of physical capital to the more advanced sector, and therefore accelerates growth even more. So higher β implies a higher growth rate from one period to the other, and a faster capital allocation to the advanced sector, which yield higher output in the steady state. Next, we shall discuss the effect of an increase in A_H on the steady state. A higher value of A_H implies that for a given output, the skill premium is higher. This in turn implies that the growth rate of high-skilled workers is higher, and therefore so is the growth rate of the physical capital allocated for the more advanced sector. This process will enhance growth even more, until eventually the economy converges into a higher steady state.

5. Productivity Differences

After analyzing the characteristics of the steady state, we can now show our main results of the paper. We analyze the transitional dynamics and show that along the transitional dynamics, as human capital accumulates, both TFP and labor productivity increase. In the next section we present another bundle of our main results in the paper that deal with the rise in the skill premium and inequality dynamics.

5.1. TFP Dynamics

A well known fact is that countries with higher human capital endowment tend to have a higher level of TFP. In the last two decades the literature for explaining this phenomenon has expanded vastly. These explanations have varied from technological differences (e.g. Romer (1993)), to non-technological differences such as social infrastructure differences (Hall and Jones, 1999) or barriers to physical capital (Restuccia, 2004). Acemoglu and Zilibotti (2001) argue that in the steady state, countries with different human capital endowments have different productivities, because of a mismatch between machines and human capital. Their paper, however, relies on the premise that human capital is exogenous and constant over time. The following proposition shows, however, that the same mechanism in which human capital attracts investment in physical capital (R&D in Acemoglu and Zilibotti (2001)) yields differences in TFP. However, since human capital is endogenous in our model, these differences exist only during these transitional dynamics.

Proposition 1. *Total factor productivity increases along time.*

Proof. TFP is given by:

$$TFP_t = \frac{Y_t}{K_t^{\alpha} \cdot \left(\frac{A_H}{A_L} H_t + L_t\right)}, \tag{19}$$

where K_t is the total amount of capital in the economy in period t. Rearranging this equation yields:

$$TFP_t = \frac{A_H \left(\frac{K_t^H}{K_t} \cdot K_t\right)^{\alpha} H_t^{1-\alpha} + A_L \left(\frac{K_t^L}{K_t} K_t\right)^{\alpha} L_t^{1-\alpha}}{K_t^{\alpha} \cdot \left(\frac{A_H}{A_L} H_t + L_t\right)^{1-\alpha}},$$

Hence, total factor productivity can be written as:

$$TFP_t = A_L \frac{\frac{A_H}{A_L} \left(S_t^H\right)^{\alpha} H_t^{1-\alpha} + \left(S_t^L\right)^{\alpha} L_t^{1-\alpha}}{\left(\frac{A_H}{A_L} H_t + L_t\right)^{1-\alpha}}, \tag{20}$$

where S_t^O is the share of capital in sector O. Next, note that from the physical capital formation and (9), that the shares of capital in sectors H and L are respectively:

$$S_t^H = \frac{\gamma H_t}{1 + H_t(\gamma - 1)}$$

and

$$S_t^L = \frac{1 - H_t}{1 + H_t(\gamma - 1)}$$

Plugging these two expressions into (20) and noting that $A_H/A_L = \gamma^{1-\alpha}$ yields:

$$TFP_t = A_L \left[\frac{(\gamma - 1)H_t + 1}{(\gamma^{1-\alpha} - 1)H_t + 1}\right]^{1-\alpha}. \tag{21}$$

It is straightforward that $\frac{\partial TFP_t}{\partial H_t} > 0$, and since H_t increases along time, so does the TFP. □

Proposition 1 proves that during the transitional dynamics, as human capital accumulates, more physical capital is allocated in the more advanced sector, a mechanism that increases the total factor productivity. However, unlike Acemoglu and Zilibotti (2001), since in the steady state countries with the same technologies and preferences have the same level of human capital, there are no productivity differences in the steady state. This mechanism also suggests another indirect channel through which human capital accelerates economic growth –through its impact on physical capital allocation and therefore on TFP measures. In this sense, our theory is close to Zeira (2009). In his model, human capital increases the profitability of adopting a new type of machines, and thus promotes economic growth. Our mechanism generalizes his result and shows that this type of mechanism is valid not only to new types of machines, but also for shifting firms from a less advanced sector to a more advanced one.

5.2. Labor Productivity Dynamics

Another productivity measure that is common in the literature is labor productivity. The following proposition states that along the transitional dynamics, countries at different stages of development, that is, with different human capital stocks have different levels of labor productivity. Following Acemoglu and Zilibotti (2001), let $\tilde{y}_t \equiv \frac{Y_t}{A_H H_t + A_L L_t}$ denote output per efficiency unit, and hence the productivity of a labor efficiency unit. Then:

Proposition 2. $\frac{\partial \tilde{y}_t}{\partial H_t} > 0$.

Proof. Output per efficiency unit is given by:

$$\tilde{y}_t = A_L \left(k_t^L\right)^{\alpha} \frac{(\gamma - 1)H_t + 1}{A_H H_t + A_L(1 - H_t)} \tag{22}$$

Differentiating this equation w.r.t. H_t yields:

$$\tilde{y}_t' = A_L \alpha \left(k_t^L\right)^{\alpha - 1} \cdot \frac{(\gamma - 1)H_t + 1}{A_H H_t + A_L(1 - H_t)} +$$
$$A_L \left(k_t^L\right)^{\alpha} \cdot \frac{(\gamma - 1)\left[A_H H_t + A_L(1 - H_t)\right] - \left[(\gamma - 1)H_t + 1\right](A_H - A_L)}{\left[A_H H_t + A_L(1 - H_t)\right]^2}.$$

The first expression in the derivative is positive. The sign of the derivative depends on the sign of the nominator in the second expression. It is straightforward that the denominator equals $A_L \gamma^{1-\alpha} (\gamma^{\alpha} - 1) > 0$. Since this expression is positive, the whole derivative is positive. □

Proposition 2 provides several insights. First, it describes the dynamics of labor productivity and shows that labor productivity differences should disappear in the long run. Suppose that an economy has an initial endowment of human capital denoted $H_0 < H^*$. Along time, human capital is accumulated, as described in (17). As a result, physical capital is shifted from the low-skilled intensive sector to the high-skilled intensive sector. This in turn implies that output per efficiency unit increases along time. However, since output per efficiency unit depends solely on the human capital stock, and since there is a unique steady state, which does not depend on initial values of human capital, in the long run labor productivity differences should disappear between countries. Second, Proposition 2 shows how the feedback effect between human capital and physical capital affects labor productivity along the transitional dynamics. In particular, it shows that investment in human capital affects the allocation of physical capital, as more physical capital is allocated in the advanced sector. This mechanism makes raw labor more productive, since skilled labor works with more physical capital, which increases the marginal productivity of the skilled labor. That is, countries with more human capital than others invest more in high-skilled labor intensive sectors, and this shift of physical capital from low-skilled intensive sectors to high-skilled intensive sectors increases the productivity of labor. This proposition, therefore, may explain why countries with different human capital endowments have different labor productivity levels, and not only TFP differences, even when human capital is taken into account.

Other Measures of Productivity. One might argue that we shall use other measures of productivity in order to analyze its dynamics. One of these measures might be to divide total output in the relative advantage that high skilled workers have in terms of marginal productivity, that is, in $MPH_t/MPL_t \cdot H_t + L_t$. Another possible measure of productivity is given by dividing output by a weighted average of TFP in the two sectors, namely by $\frac{A_H}{A_L} H_t + L_t$. Appendix B shows in detail that these measures of labor productivity do not change the result of Proposition 2.

6. Skill Premium and Inequality Dynamics

After discussing in the previous section the dynamics of productivity, we discuss in this section the dynamics of the skill premium and income inequality. It is a well known fact that in the last few decades many economies have experienced both a rise in the skill premium and a rise in income inequality accompanied with a rise in the educated labor force. The main two explanations for the coincidence of the three phenomena were skill-biased technological change (Galor and Moav, 2000; Acemoglu, 1998) and capital-skill complementarity (Krusell et al., 2000). In this section we provide another possible explanation for these phenomena. In particular, we show that along the transitional dynamics the skill premium increases, and that income inequality increases in the beginning of the development process, but may exhibit a Kuznets curve pattern. Let $w_t^H - w_t^L$ be the skill premium. Then:

Proposition 3. *Along the transitional dynamics the skill premium increases.*

Proof. The skill premium equals:

$$w_t^H - w_t^L = (1-\alpha) A_L (\gamma - 1) \left(k_t^L\right)^\alpha .$$

The only element that evolves along time is k_t^L, which increases along time. Consequently the skill premium increases as well. $\square$

Proposition 3 suggests that along the transitional dynamics the skill premium increases, despite the rise of the high-skilled labor force and the decline of the low-skilled labor force.The reason for this result is the above mentioned mechanism. During the transitional dynamics, both wages –of low-skilled- and high-skilled labor –increase. The increase in the wage of low-skilled labor is a consequence of a decrease in the supply of low-skilled labor and an increase in the demand for low-skilled labor, which is the result of allocating more physical capital (per worker) in this sector than in the previous period. The increase in the wage of the high-skilled labor is a consequence of an increase in the demand for high-skilled labor due to an increase in the physical capital allocated for this sector. The rise in the demand for high-skilled labor offsets the negative effect that the increase in the supply of high-skilled labor has on the wage of the high skilled workers, so their wage increases as well. Note that the increase in the physical capital per worker in sector H is larger than the increase in the capital per worker allocated for sector L, since $k_t^H = \gamma k_t^L$. This relatively large increase in the wage of high-skilled labor offsets the negative effect on the skill premium that the increase in the wages of the low-skilled labor has.

The result about the skill premium assists us to explore the dynamics of income inequality in the economy. We measure income inequality by the variance of income in the economy. The following proposition shows that the variance of income increases for small values of H_t and declines for sufficiently high values of H_t. Since H_t increases along time we conclude that income inequality rises at the outset of development, and may decline at later stages of development. Hence, income inequality may increase along time, or exhibit a Kuznets curve pattern.

Proposition 4. *At the beginning of the development process, the variance of income increases as H_t increases. At later stages of development, the variance of income may decrease.*

Proof. The average income at period t is given by:

$$\overline{w}_t = H_t \cdot w_t^H + (1 - H_t) \cdot w_t^L.$$

The variance of income is given by:

$$\sigma_t^2 = H_t \cdot \left(w_t^H - \overline{w}_t\right)^2 + (1 - H_t)\left(\overline{w}_t - w_t^L\right)^2 = H_t(1 - H_t)(w_t^H - w_t^L)^2, \quad (23)$$

where the last expression is obtained after substituting into the first expression

the wages as given by (3) and (4). Differentiating the last expression with respect to H_t yields:

$$\frac{\partial \sigma_t^2}{\partial H_t} = (1 - 2H_t)(w_t^H - w_t^L)^2 + 2H_t(1 - H_t)(w_t^H - w_t^L)\frac{\partial (w_t^H - w_t^L)}{\partial H_t}.$$

Using (23), this equation becomes:

$$\frac{\partial \sigma_t^2}{\partial H_t} = [(1-\alpha)A_L(\gamma-1)(k_t^L)^\alpha]^2(3-4H_t),$$

where the last expression is obtained by using (15). The derivative is positive as long as $0 < H_t < 0.75$, and negative as long as $0.75 < H_t < 1$. Since H_t increases along time, the variance in income increases at the outset of the development process, and decreases at later stages of the development process. □

Proposition 4 sheds light on the dynamics of income inequality in the economy. As the economy develops, two forces with opposite signs affect income inequality: wage inequality between the two groups of workers (the skill premium) and the relative abundance of high-skilled workers. As the economy develops the skill premium increases (as shown in Proposition 3), a force that increases inequality, but the relative abundance of high-skilled workers increases as well, which in turn decreases income inequality. According to Proposition 4, at the outset of the development process, the former is greater than the latter, while in later stages of development the opposite may occur. In later stages of development, it is the relative equality between high-skilled workers that may dominate the rising inequality between the two groups of workers. Note that this result is not a trivial outcome of the fact that the factor $H_t(1-H_t)$ has an inverse-U shape, as occurs in models where wages are exogenous. Here with the endogenous determination of wages, and the Inada condition of the production functions, the third element in RHS of (23) falls at an infinite rate at the vicinity of $H_t = 0$, which, in a single sector model, dominates the rise in $H_t(1-H_t)$ in that vicinity.[6]

Note that it might be that parameter values are such that the economy converges to a steady state in which $H^* < 0.75$. In this case, the economy experiences a growing inequality along time. In the second case, where the economy converges to a steady state with $H^* > 0.75$, income inequality displays a Kuznets curve pattern, as it rises at the outset of the development process and declines afterwards. The parameters that yield a Kuznets curve pattern are the ones that yield a higher high-skilled labor stock in the steady state. That is, higher values of β, A_H and A_L.

7. Conclusions

We presented a general equilibrium dynamic model with two sectors –one more productive than the other. We used this model to analyze how the heterogeneity in sector productivity affects the dynamics of physical and human

[6]See, for example, Maoz & Moav (1999), where in a model with a single sector, inequality is monotonically falling for all positive H_t., as one example of many for the dominance of the wage effect induced by the Inada conditions.

capital accumulation, skill premium, income inequality, labor productivity and total factor productivity.

The analysis of the resulting macroeconomic equilibrium highlights a feedback relationship between investment in education and investment of physical capital in the advanced sector, as one promotes the marginal productivity of the other. This feedback mechanism leads to one of our main results, namely that the skill premium rises over time. We also find that the rising skill premium may lead to a dynamic pattern of a rising income inequality. However, it is also possible that income inequality shall not be monotonically rising, but instead, shall exhibit Kuznets curve dynamics, in which it initially rises and from a certain point in time begins to decline, even though the skill premium continues to rise. The decline in inequality occurs when the number of high-skilled workers is sufficiently large to make the relative equality within the high-skilled workers dominate the rising inequality between the two groups of workers.

This feedback mechanism also highlighted another indirect channel through which human capital promotes economic growth. As human capital is accumulated, more physical capital is allocated to the more advanced sector, and thus output grows faster. This effect leaded to the rise in the TFP and labor productivity along time. We showed that in this sense our results are close in their nature to Acemoglu and Zilibotti (2001), Caselli and Coleman (2006) and Zeira (2009), only in our model we analyze the transitional dynamics and not merely the steady state equilibrium. We also added to their results and used the model for analyzing inequality pattern. These results about the rise in the skill premium, income inequality and productivity fit the empirical findings presented by a massive body of literature.

The model is of a closed Overlapping Generations framework with some specific assumptions. Hence, the issue of robustness should be discussed. First, similar specific functional forms are widespread in this strand of the literature. Second, the functional forms are motivated strongly by empirical evidence. Using another production functions and a utility function that satisfy the usual assumptions will change the results quantitatively but not qualitatively.

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Appendix A

In this Appendix we provide a proof for Lemma 2. More generally, we provide a proof for a more general lemma, of which Lemma (2) is a private case:

Lemma 4. *Let $f(x)$ be the following function:*

$$f(x) = A \cdot x^{1+\alpha} + B \cdot x^{\alpha}, \tag{A.1}$$

where A, B and α satisfy $A > 0, B > 0, 0 < \alpha < 1$, and let n be a positive integer. Then $\forall x > 0$:
(i) $f^{(n)} < 0$ if n is even.
(ii) $f^{(n)} > 0$ if n is odd.

Proof:. Differentiating the function $f(x)$ yields:

$$f'(x) = A \cdot (1+\alpha) \cdot x^{\alpha} + B \cdot \alpha \cdot x^{\alpha-1}. \tag{A.2}$$

So (ii) holds for $n = 1$. Higher order differentiations of $f(x)$ yield:

$$f''(x) = A \cdot (1+\alpha) \cdot \alpha x^{\alpha-1} + B \cdot \alpha \cdot (\alpha - 1) \cdot x^{\alpha-2} \tag{A.3}$$

By standard induction, the n^{th} derivative of $f(x)$ satisfies:

$$f^{(n)}(x) = A \cdot (\alpha+1) \cdot \left[\prod_{j=0}^{n-2} (\alpha - j)\right] \cdot x^{\alpha-n+1} + B \cdot \left[\prod_{j=0}^{n-2} (a - j)\right] (\alpha - n + 1) \cdot x^{\alpha-n} \tag{A.4}$$

From (A.4) it follows that for all $n \geq 2$:

$$\lim_{x\to\infty} f^{(n)}(x) = 0. \tag{A.5}$$

Next, note that (A.4) can be rearranged as follows:

$$f^{(n)}(x) = \left[\prod_{j=0}^{n-2} (\alpha - j)\right] \cdot [A \cdot (\alpha+1) \cdot x + B \cdot (\alpha - n + 1)] \cdot x^{\alpha-n}. \tag{A.6}$$

The term in the left brackets is positive if n is even and negative if n is odd. Note also that for each n there exist $\overline{x}(n)$ such that $\forall x < \overline{x}(n)$, the term in the right brackets is negative. $\overline{x}(n)$ is given by:

$$\overline{x}(n) = \frac{B \cdot (n - \alpha - 1)}{A \cdot (\alpha + 1)}. \tag{A.7}$$

It is straightforward that

$$\lim_{n\to\infty} \overline{x}(n) = \infty. \tag{A.8}$$

Hence, $\forall x > 0$, if n is sufficiently large, then the term in the right brackets is negative. Thus, $\forall x > 0$ there exists a certain even $\overline{n}$ that is sufficiently large such that

$$f^{(n)} < 0. \tag{A.9}$$

Due to (A.5) and (A.9), this $\overline{n}$ satisfies:

$$f^{(n-1)} < 0. \tag{A.10}$$

Likewise, due to (A.5) and (A.10):

$$f^{(n-2)} > 0, \tag{A.11}$$

and so on, implying that $f^{(n)}(x) < 0$ for each even n and that $f^{(n)}(x) > 0$ for each odd n as long as $n \geq 2$, which is mandatory for (A.5) to hold. $\square$

Since the function $Y_t(H_t)$ as given by (17) is of the form as in (A.1), $Y_t''(H_t) < 0$.

Appendix B

The following appendix shows in detail how different measures of labor productivity do not change the main result of Proposition 2 that the transitional dynamics labor productivity increases. One suggested measure is to divide output by $MPH_t/MPL_t \cdot H_t + L_t$. I now show that this measure does not change the result.

Since factor markets are competitive, $MPH_t = w_t^H$ and $MPL_t = w_t^L$. It is straightforward that

$$\frac{w_t^H}{w_t^L} = \frac{(1-\alpha)A_H \left(\gamma k_t^L\right)^\alpha}{(1-\alpha)A_L \left(k_t^L\right)^\alpha} = \gamma$$

Let $\tilde{\tilde{y}}_t \equiv \frac{Y_t}{\frac{MPH_t}{MPL_t}H_t+(1-H_t)}$ denote the other measure of productivity. Then:

$$\tilde{\tilde{y}}_t = \frac{A_H H_t \left(\gamma k_t^L\right)^\alpha + A_L(1-H_t)\left(k_t^L\right)^\alpha}{\gamma H_t + 1 - H_t} = \left(k_t^L\right)^\alpha \frac{A_H H_t (\gamma)^\alpha + A_L(1-H_t)}{(\gamma-1)H_t+1}.$$

Differentiating $\tilde{\tilde{y}}_t$ w.r.t. H_t yields:

$$\begin{aligned} \tilde{\tilde{y}}_t' &= \alpha \left(k_t^L\right)^{\alpha-1} \cdot \frac{\partial k_t^L}{\partial H_t} \frac{A_H H_t \gamma^\alpha + A_L(1-H_t)}{(\gamma-1)H_t+1} \\ &+ \left(k_t^L\right)^\alpha \frac{(A_H\gamma^\alpha - A_L)\cdot[(\gamma-1)H_t+1] - [A_H\gamma^\alpha H_t + A_L(1-H_t)](\gamma-1)}{[(\gamma-1)H_t+1]^2} \\ &= \alpha \left(k_t^L\right)^{\alpha-1} \cdot \frac{\partial k_t^L}{\partial H_t} \frac{A_H H_t \gamma^\alpha + A_L(1-H_t)}{(\gamma-1)H_t+1} > 0. \end{aligned}$$

Note that the nominator of the second part of the derivative equals zero, and therefore this part of the derivative equals zero. Since $\frac{\partial k_t^L}{\partial H_t} > 0$, the derivative is greater than zero. This implies that this measure of productivity does not change our results.

Another measure might be obtained by dividing output in $A_H/A_L H_t + 1 - H_t$. In this case, let $\hat{y}$ denote:

$$\hat{y}_t = \frac{A_H H_t \left(\gamma k_t^L\right)^\alpha + A_H H_t \left(k_t^L\right)^\alpha}{\frac{A_H}{A_L}H_t + 1 - H_t} = A_L (k_t^L)^\alpha \cdot \frac{A_H \gamma^\alpha H_t + A_L(1-H_t)}{A_H H_t + A_L(1-H_t)} = A_L \tilde{y}_t.$$

Since A_L is constant along time, $\hat{y}_t$ behaves exactly as $\tilde{y}_t$ along time, and the result concerning productivity differences between countries with different human capital endowments still holds.